**Cost-constrained multi-depot vehicle routing problem formulation**

This document details the formulation for solving a cost-constrained multi-depot vehicle routing problem with jobs, depots and crews. Note that since there are jobs and depots, there are a total of locations. The objective of this formulation is to maximise the job score balanced by the distance travelled.

Although this formulation allows for any crew to attend to any job, the formulation can be altered to accommodate for restrictions on crews only allowed to attend to specific jobs. Furthermore, to increase solving speed, it may be helpful to include hard constraints for “must-do” jobs for the day.

The solving time/complexity grows exponentially if any of the parameters , or increases. Thus considerable care should be taken when deciding on how to adjust the model to real world requirements.

**Sets**

* : Set of jobs
* : Set of crews
* : Set of depots
* : Set of all locations
* : Set of jobs that are due today

**Constants**

|  |  |
| --- | --- |
| **Constant description** | **Notation** |
| Job score | where |
| Job due time | where |
| Job estimated completion time | where |
| Penalty score for job | where |
| Time from location to location using crew  *is only relevant for distances that involve the depots)* | where and |
| Start/finish time for truck. (Shift time) | where |
| Start/finish depot for truck. | where |
| Set of jobs a crew can do (Crew type) |  |

**Decision variables**

**N.B**: The X matrix is not symmetrical. The origins (Rows) include the start depot at the first index, while the destinations (Columns) include the end depot at the last index. That is, index 0 is the start depot *for that truck*, while index n+1 is the end depot *for that truck*. For brevity, we drop the subscript k from 0k and (n+1)k.

**Objective**

Note: Beware of indexing in Julia anytime there is indexing with regards to a matrix. Positional index may be different to location index, due to the fact that the travel matrices do not have the end depot as an origin, and the start depot as a destination.

**Constraints**

1. At most one crew for each job
2. Vehicle leaving and returning (Perhaps add SOS, where sum is less than or equal to one)
3. Conservation of flow constraint (Enter must leave)
4. Cannot visit itself (Covered by subtour constraints, model tightening)
5. Temporal constraints (Ensure a valid tour)
6. Shift time constraints. Link between job time and crew shift times
7. Penalty constraints

for

for

1. Crew type constraints
2. Shift code constraints

Optional constraint: Enforce the time of non-visited jobs to be 0